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# THE FINE-TUNING PROBLEM OF THE ELECTROWEAK SYMMETRY BREAKING MECHANISM IN MINIMAL SUSY MODELS\*

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## Abstract

We calculate the region of the MSSM parameter space (i.e.  $M_{1/2}$ ,  $m_0$ ,  $\mu$ , ...) compatible with a correct electroweak breaking and a realistic top-quark mass. To do so we have included *all* the one-loop corrections to the effective potential  $V_1$  and checked their importance in order to obtain consistent results. We also consider the fine-tuning problem due to the enormous dependence of  $M_Z$  on  $h_t$  (the top Yukawa coupling), which is substantially reduced when the one-loop effects are taken into account. We also explore the reliability of the so-called "standard" criterion to estimate the degree of fine-tuning. As a consequence, we obtain a new set of upper bounds on the MSSM parameters or, equivalently, on the supersymmetric masses perfectly consistent with the present experimental bounds.

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# 1 Introduction

The Minimal Supersymmetric Standard Model (MSSM), is characterized by a Lagrangian

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \quad (1)$$

where  $\mathcal{L}_{SUSY}$  is the supersymmetric part (derived from  $W_{obs}$ , the observable superpotential which includes the usual Yukawa terms  $W_Y$  plus a mass coupling between the two Higgs doublets,  $\mu H_1 H_2$ ) and  $\mathcal{L}_{soft}$  contains the SUSY breaking terms and is given at the unification scale  $M_X$  by:

$$\mathcal{L}_{soft} = -m_0 \sum_{\alpha} |\phi_{\alpha}|^2 - \frac{1}{2} M_{1/2} \sum_{a=1}^3 \bar{\lambda}_a \lambda_a - (A m_0 W_Y + B m_0 \mu H_1 H_2 + h.c.). \quad (2)$$

Here  $m_0$  and  $M_{1/2}$  are the universal soft breaking masses (evaluated at  $M_X$ ) for scalars ( $\phi_{\alpha}$ ) and gauginos ( $\lambda_a$ ) respectively;  $A$  and  $B$  stand for the trilinear and bilinear couplings between scalar fields. So all the supersymmetric masses are fixed once we have chosen values for the following MSSM parameters:  $m_0$ ,  $M_{1/2}$ ,  $\mu$ ,  $A$ ,  $B$ ,  $h_t$ , where  $h_t$  is the top Yukawa coupling (we are neglecting the influence, small in our case, of the bottom and tau Yukawa couplings).

In particular this set of parameters gives us the form of the Higgs potential in the MSSM which is responsible for the electroweak breaking process<sup>1</sup>. By imposing the correct electroweak breaking scale and a reasonable top-quark mass, the allowed region of values for these parameters is considerably restricted. Furthermore, if one also requires the absence of fine-tuning in the value of  $h_t$  through the ordinary equation<sup>2</sup>

$$\frac{\delta M_Z^2}{M_Z^2} = c \frac{\delta h_t^2}{h_t^2}, \quad (3)$$

by setting an upper bound for  $c$ , the allowed values for the parameters are more constrained.

Here we present an analysis of these issues following the recent one done by Ross and Roberts<sup>3</sup>, but refined with the inclusion of the one-loop corrections to the effective Higgs potential<sup>4</sup> (theirs was done considering only the renormalization-improved tree-level potential  $V_0$ ), which gives substantially different results.

## 2 Radiative electroweak breaking

The one-loop Higgs potential of the MSSM,  $V_1(Q)$ , is given at a scale  $Q$  by the sum of two terms: the commonly used renormalization-improved tree-level potential,

$$V_0(Q) = \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + h.c.), \quad (4)$$

with  $m_i^2 = m_{H_i}^2 + \mu^2$ ,  $i = 1, 2$  ( $m_{H_i}^2(M_X) = m_0^2$ ) and  $m_3^2 = Bm_0\mu$ ; and the one-loop corrections<sup>5</sup>,

$$\Delta V_1(Q) = \frac{1}{64\pi^2} Str \left[ \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right]. \quad (5)$$

Here  $\mathcal{M}^2$  is the field dependent tree-level squared mass matrix which contains *all* the states of the theory properly diagonalised, thus including the dependence on  $H_1$  and  $H_2$ ). All the parameters appearing in these equations are evaluated at some scale  $Q$  and run with it; they can be computed by solving the standard RGEs<sup>6</sup>, using the present values for the gauge couplings and taking into account the supersymmetric thresholds.

To study electroweak breaking we minimize  $V_1$  to obtain  $v_1 \equiv \langle H_1 \rangle$  and  $v_2 \equiv \langle H_2 \rangle$ . An example of our results can be seen in Fig. 1: both  $V_0$  and  $V_1$  predict electroweak breaking but for different values of  $v_1$  and  $v_2$ . From our analysis we see that: *i*) the tree-level approximation is not reliable, *ii*) the top-stop approximation is not accurate enough to stand for the whole one-loop corrections and *iii*) the complete one-loop solutions are much more stable versus  $Q$ .

However we may find that, for some values of the parameters, this stability is partially spoiled in the region of electroweak breaking ( $Q \sim M_Z$ ) due to large logarithmic corrections. In order to give general results we choose to take  $v_1(Q)$  and  $v_2(Q)$  at some scale  $\hat{Q}$  where  $\Delta V_1$  is negligible<sup>4,7</sup> and then perform the wave function renormalization of the Higgs fields from  $\hat{Q}$  to  $M_Z$  (which is a small effect indeed).

We can now calculate  $M_Z$  as

$$(M_Z^{phys})^2 \simeq \frac{1}{2}(g^2(Q) + g'^2(Q))[v_1^2(Q) + v_2^2(Q)] \Big|_{Q=M_Z^{phys}} \quad (6)$$

and constrain the MSSM parameters by requiring: *a*) correct electroweak breaking (i.e.  $M_Z^{phys} = M_Z^{exp}$ ), *b*) reasonable top-quark mass and *c*) absence of electric charge and colour breakdown<sup>6</sup>. The resulting region of allowed values is enhanced and displaced from the one obtained in Ref. 3 (see Fig. 2). We have also evaluated the effect of varying  $A$ ,  $B$  and  $|\mu/m_0|$  with similar results<sup>8,9</sup>.

### 3 The fine-tuning problem

In the previous section we have restricted the possible values of the MSSM parameters by imposing a correct electroweak breaking. But we still can find arbitrarily high values of these parameters compatible with this constraint, which would lead us to a problem of fine-tuning<sup>3</sup>. In particular we are interested in the parameter to which  $M_Z$  is most sensitive, that is  $h_t$ , and the degree of fine-tuning is given by Eq. 3 in which  $c$  depends on the whole set of MSSM parameters. So avoiding fine-tuning means setting an upper bound on  $c$ , e.g.  $c \lesssim 10$  as in Ref. 3 (see Fig. 2a). In our case, the inclusion of the one-loop corrections soften the dependence of  $M_Z$  on  $h_t$ , giving for the same bound,  $c \lesssim 10$ , a broader region of allowed parameters.

But the standard criterion of fine-tuning, Eq. 3, is *ambiguously* defined as it depends on *i*) the independent parameters of the theory and *ii*) the physical quantity we are fitting (note that taking  $h_t$  ( $M_Z$ ) instead of  $h_t^2$  ( $M_Z^2$ ) would change  $c$  into  $2c$  ( $c/2$ )). Moreover we see that it measures sensitivity rather than fine-tuning: a relationship extremely sensitive between  $M_Z$

and  $h_t$  could lead to values of  $c$  always higher than the bound, making this criterion meaningless. We have checked that fortunately this is not the case in the MSSM<sup>8</sup>.

From all these considerations we see that the standard fine-tuning criterion (3) is more qualitative than quantitative, so we should conservatively relax the former bound<sup>3</sup> up to  $c \lesssim 20$  at least. This leads us to new upper limits on the MSSM parameters, as can be seen in Fig. 2b:  $m_0, \mu \lesssim 650$  GeV and  $M_{1/2} \lesssim 400$  GeV, that imply upper bounds on the sparticle spectrum:

$$\begin{aligned} \text{Gluino} &: M_{\tilde{g}} \lesssim 1100 \text{ GeV} \\ \text{Lightest chargino} &: M_{\chi^\pm} \lesssim 250 \text{ GeV} \\ \text{Lightest neutralino} &: M_\lambda \lesssim 200 \text{ GeV} \\ \text{Squarks} &: m_{\tilde{q}} \lesssim 900 \text{ GeV} \\ \text{Sleptons} &: m_{\tilde{l}} \lesssim 450 \text{ GeV} \end{aligned}$$

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## Figure captions

**Figure 1:**  $v_1$  and  $v_2$  versus the  $Q$  scale between  $M_Z$  and 2 TeV (in GeV), for  $m_0 = \mu = 100$  GeV;  $M_{1/2} = 180$  GeV;  $A = B = 0$ ;  $h_t = 0.250$ . Solid lines: complete one-loop results; dashed lines: "improved" tree-level results; dotted lines: one-loop results in the top-stop approximation.

**Figure 2:** The case  $A = B = 0$ ,  $|\mu/m_0| = 1$  with (a) the tree-level potential  $V_0$  and (b) the whole one-loop effective potential  $V_1$ . Diagonal lines correspond to the extreme values of  $m_t$  as were calculated in Ref. 3:  $m_t = 160, 100$  GeV. Transverse lines indicate constant values of  $c$ .